Solving Multiple-Angle Trig Equations + Review (3.10+3.12) Homework

1. The function f is given by $f(x) = 3\sin(2x) - 4$. Find all input values in the domain of f that yield an output value of $-\frac{5}{2}$.

2. The function g is given by $g(x) = \sqrt{2}\sec\left(x - \frac{\pi}{4}\right)$. Find all values in the domain of g that yield an output value of 2.

3. The function h is given by $h(x) = 3\tan\left(\frac{1}{3}x\right)$. Find all input values in the interval $[-\pi, \pi]$ that yield an output value of $\sqrt{3}$.

4. The function k is given by $k(x) = 2\cos\left(x + \frac{4\pi}{3}\right) + 3$. Find all input values in the interval $[0, 2\pi)$ that yield an output value of 1.

5. Which one of the following represents the x-values at which the graph of $f(x) = -\csc\left(2x - \frac{\pi}{3}\right) + 4$ has vertical asymptotes?

   A) $x = \frac{\pi}{6} + \frac{\pi}{2}k$, where $k \in \mathbb{Z}$

   B) $x = \frac{\pi}{6} + \pi k$, where $k \in \mathbb{Z}$

   C) $x = \frac{\pi}{6} + 2\pi k$, where $k \in \mathbb{Z}$

   D) $x = \frac{\pi}{6} + 4\pi k$, where $k \in \mathbb{Z}$

6. Which one of the following represents the x-values at which the graph of $g(x) = \tan\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$ has vertical asymptotes?

   A) $x = -\frac{\pi}{2} + \pi k$, where $k \in \mathbb{Z}$

   B) $x = -\frac{\pi}{2} + 2\pi k$, where $k \in \mathbb{Z}$

   C) $x = \frac{\pi}{2} + 2\pi k$, where $k \in \mathbb{Z}$

   D) $x = \frac{\pi}{2} + 4\pi k$, where $k \in \mathbb{Z}$

7. The function f is given by $f(x) = 2\cos(3x) + 4\sqrt{3}$. Find all input values in the domain of f that yield an output value of $5\sqrt{3}$.

8. The function f is given by $f(\theta) = -\frac{\tan^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$. Express this function as an expression in terms of $\cos\theta$ and with no other trigonometric function involved.

9. The function h is given by $h(\theta) = \frac{\csc\left(\frac{\pi}{2} - \theta\right)}{\tan(-\theta)}$. Express this function as an expression in terms of $\csc\theta$ and with no other trigonometric function involved.

10. Based on FRQ4(C) from the 2024 AP Exam:
    The function g is given by
    $$g(x) = \tan^{-1}(\sin(2x))$$
    Find all values in the domain of g that yield an output value of $\frac{\pi}{4}$.

11. When graphed in standard position, the terminal ray of an angle of measure θ intersects a circle of radius 13 at the point $(-5,12)$.
    a. Find $\sin\theta$
    b. Find $\sec\theta$
    c. Find $\tan\theta$
    d. Find $\cos(2\theta)$
    e. Find $\sin\left(\theta + \frac{3\pi}{4}\right)$

12. When graphed in standard position, the terminal ray of an angle of measure $\frac{7\pi}{6}$ intersects a circle of radius 10 at what coordinates?

Evaluate without the use of a calculator by first rewriting in an equivalent form using a trigonometric identity.

13. $10\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)$

14. $4\cos^2\left(\frac{\pi}{12}\right) - 4\sin^2\left(\frac{\pi}{12}\right)$

15. $2\cos^2\left(\frac{\pi}{12}\right) - 5$

16. $6\cos^2\left(\frac{\pi}{5}\right) + 6\sin^2\left(\frac{\pi}{5}\right)$

Solving Multiple-Angle Trig Equations HW Solutions

1. $x = \frac{\pi}{12} + \pi k$ or $x = \frac{5\pi}{12} + \pi k$, where k is any integer
2. $x = \frac{\pi}{2} + 2\pi k$ or $x = 2\pi k$, where k is any integer
3. $x = \frac{\pi}{2}$
4. $x = \frac{5\pi}{3}$
5. A) $x = \frac{\pi}{6} + \frac{\pi}{2}k$, where k is any integer
6. B) $x = -\frac{\pi}{2} + 2\pi k$, where k is any integer
7. $x = \frac{\pi}{18} + \frac{2\pi}{3}k$; $x = \frac{11\pi}{18} + \frac{2\pi}{3}k$, where k is any integer
8. $f(\theta) = -\frac{1}{\cos^4\theta}$
9. $h(\theta) = -\csc\theta$
10. $x = \frac{\pi}{4} + \pi k,\ k \in \mathbb{Z}$
11.
    a. $\sin\theta = \frac{12}{13}$
    b. $\sec\theta = -\frac{13}{5}$
    c. $\tan\theta = -\frac{12}{5}$
    d. $\cos(2\theta) = -\frac{119}{169}$
    e. $\sin\left(\theta + \frac{\pi}{4}\right) = \frac{-17\sqrt{2}}{26}$
12. $\left(-5\sqrt{3}, -5\right)$
13. $10\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right) = 5\left(2\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)\right) = 5\left(\sin\left(2\cdot\frac{\pi}{12}\right)\right) = 5\sin\left(\frac{\pi}{6}\right) = \frac{5}{2}$
14. $4\cos^2\left(\frac{\pi}{12}\right) - 4\sin^2\left(\frac{\pi}{12}\right) = 4\left(\cos^2\left(\frac{\pi}{12}\right) - \sin^2\left(\frac{\pi}{12}\right)\right) = 4\cos\left(2\cdot\frac{\pi}{12}\right) = 4\cos\left(\frac{\pi}{6}\right) = 2\sqrt{3}$
15. $2\cos^2\left(\frac{\pi}{12}\right) - 5 = \left(2\cos^2\left(\frac{\pi}{12}\right) - 1\right) - 4 = \cos\left(2\cdot\frac{\pi}{12}\right) - 4 = \cos\left(\frac{\pi}{6}\right) - 4 = \frac{\sqrt{3}}{2} - 4$
16. $6\cos^2\left(\frac{\pi}{5}\right) + 6\sin^2\left(\frac{\pi}{5}\right) = 6\left(\cos^2\left(\frac{\pi}{5}\right) + \sin^2\left(\frac{\pi}{5}\right)\right) = 6(1) = 6$